(Continued from page 713)

INFORMATION RETRIEVAL

Key Words: Sphere-5, Carboxymethyl Cellulose-5, Drag Force-8, Drag Coefficient-8, Variational-10, Non-Newtonian-, Ostwald-De Waele-, Power Law-, Rheology-.

Abstract: A first approximation to the drag force on a sphere moving slowly through an Ostwald-de Waele fluid is obtained by means of a variational principle proposed by Pawlowski. This is the best available description of the drag force for $n > 0.76$; the corrected zeroth approximation of Tomita is superior for $n < 0.76$. Previously available experimental data are reanalyzed and compared with these developments. A zeroth approximation is given for a Sisko fluid, but no appropriate experimental data have been reported.

Reference: Slattery, John C., **A.I.Ch.E. Journal, 8,** No. 5, p. 663 (November, 1962).

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surface of decomposition of a spherical particle, Equation (3) represents the most general form of the expression for the instantaneous reaction rate. A more detailed investigation will appear elsewhere at an early date (*6*).

NOTATION

D = diffusion coefficient in equivalent film, sq. ft./hr.
D_e = effective diffusion coefficient in porous layers, sq. ft./hr.
k_g = mass transfer coefficient, lb./hr. sq. ft. atm. $= D/R'T \cdot Z_g$
k_r = reaction rate constant, lb./hr. sq. ft. atm.
k_d = diffusion constant, (g./cm. mt. atm.)
K = thermodynamic equilibrium constant
K_M = modified Nusselt number $= \dfrac{k_g \cdot R' \cdot T \cdot R}{D_e}$
p_{H_2O} = equivalent partial pressure of water at the interface, atm.
p_i = partial pressure of hydrogen at the interface, atm.
p_o = partial pressure of hydrogen in the bulk gas
p_e = equilibrium partial pressure of reducing gas, atm.
$Q = k_r$ = reaction rate constant
r' = dimensionless parameter, (r/R)
r = radial position of decomposition interface, ft.
R = radius of undecomposed sphere, ft.
R' = gas constant
S = mole ratio of oxygen/iron in particle $= r'^3$
T = average temperature in the system, °R.
W_o = weight loss of oxide when fully reduced, g.
α = stoichiometric constant
ρ = density of original oxide, lb./cu. ft.
θ = time of decomposition, hr.

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An Analysis of Blow Over and the Steady State Bed Particle Size Distribution in a Reacting Fluid Bed

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There are fluid bed reactor situations in which the bed particles are attacked, chemically or physically, resulting in a decrease in particle size with time. Under such conditions it is necessary to feed bed material continuously to make up for losses resulting from blow over and vapor by reaction. As a consequence the steady state particle size distribution in the bed, the mass rate of blow over, and the particle size distribution in the blow over must be known. The first influences the quality of fluidization, the second determines the required capacity of the collection system, and the last will influence the type of collection system which is required.

An understanding of the behavior of fluidized beds has been the object of considerable work during the past twenty years. However much of this has been devoted to an understanding of the physical characteristics of the bed, that is gas flow patterns, particle flow patterns, and the mixing processes important within the bed. Recent work on the entrainment of particles from the bed by mechanical action of the gas usually relates entrainment to the freeboard above the bed via the exponential equation involving an entrainment constant. The work of Lewis et al. (*2*) is such a study. Fluidized bed technology has been reviewed in the text by Zenz and Othmer (*4*), in which a chapter is devoted to the process of physical entrainment.

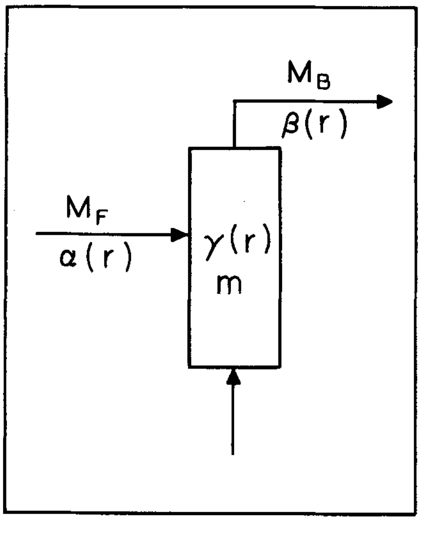

Fig. 1. Schematic of the model.

The recent contribution of Yagi and Kunii (*3*) to the understanding of particle behavior/conversion in fluidized beds is of interest. The authors present a technique for estimating the mean residence time of solids in the stream and present an elutriation velocity constant which, if it can be evaluated for situations of practical interest, permits the estimation of the average residence time of particles in the bed. However the methods for prediction of this constant for an unknown situation are not clearly so well defined as methods for its estimation for an existing and operating reactor. Furthermore a necessary assumption for the use of the techniques presented is that the particle diameter does not change during the reaction. This assumption is not acceptable for the problem described below.

There has been little if any work on the behavior of fluidized beds as chemical reactors. The author knows of none devoted to the problem of the steady state bed particle size distribution and the carry-over or blow over resulting from particle size reduction by chemical reaction to a point at which they are physically carried by the gas as a part of the escaping gas stream.

This paper is concerned with the later problem. A chemical reaction is assumed in progress causing a reduction in particle size permitting the gas to pick up and carry over bed particles which have been reduced to such a size that they move as an integral part of the gas stream.

STATEMENT OF THE PROBLEM

The system is shown schematically in Figure 1. The fluid bed particles are assumed to be attacked chemically (and/or physically) by the gas stream causing a reduction in size. The steady state particle size distribution in the fluidized bed is the desired quantity. The feed rate of solid, the particle size distribution in the feed, all geometry factors related to bed construction, gas velocities, and the surface reaction kinetics are presumed known for the present. The blow over rate and the particle size distribution in the blow over are additional unknowns to be calculated if possible. Since the bed is fluidized, it seems reasonable to assume that $\gamma(r)$ is not dependent on position within the bed. This limits the results to steady state particulate fluidization.

An interval balance on r yields

$$\frac{d}{dr}[\overline{R}(r)\cdot\gamma(r)] - \frac{A_e\,v(r)}{V\overline{R}(r)}[\overline{R}(r)\cdot\gamma(r)] = -M_F\frac{\alpha(r)}{V} \qquad (1)$$

For which the solution is well known to be

$$\gamma(r) = K_2\int_r^\infty \alpha(\omega)\, e^{-\int_r^\omega \frac{A_e v(\zeta)}{V\overline{R}(\zeta)}d\zeta}\, d\omega \qquad (2)$$

wherein

$$K_2 = \frac{M_F}{V\overline{R}(r)} \qquad (3)$$

and

$$[\overline{R}(r)\cdot\gamma(r)] \rightarrow 0 \qquad (4)$$
$$\text{as } \gamma(r) \rightarrow \infty$$

has been taken as the most logical boundary condition. The evaluation of

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INFORMATION RETRIEVAL

Key Words: Solubility-8, Correlation-8, Hydrogen Bonding-8, Heat of Mixing-8, Monochlorodifluoromethane-1, Dichloromonofluoromethane-1, Triethylene Glycol Dimethyl Ether-1, Tetraethylene Glycol Dimethyl Ether-1, N,N-Dimethylacetamide-1, N,N-Dimethylformamide-1, Ethyllaurate-1, Diethyloxalate-1, Diethyladipate-1, 1,3-Butandiol-1, 1,4-Butandiol-1, Solutions-2, Temperature-6, Composition-6, Pressure-7, Heat of Mixing-7, Functionality-7.

Abstract: Hydrogen bonding is believed to be primarily responsible for the observed high solubilities of the halogenated methanes in organic solvents. Three solvents, tetraethylene glycol dimethyl ether (best on mole basis), dimethylacetamide, and dimethylformamide (best on weight basis), are excellent solvents for monochlorodifluoromethane and dichloromonofluoromethane. A method accurate within 10% is presented for calculating heats of mixing from solubility data. Analysis of the data in terms of activities and activity coefficients utilizing the approach of Mastrangelo was found to be a good method for correlating the data and testing for experimental consistency.

Reference: Albright, Lyle F., Paul T. Shannon, Francois Terrier, and Ping Lin Chueh, **A.I.Ch.E. Journal, 8,** No. 5, p. 668 (November, 1962).

Key Words: Drops (Droplets)-8, Size-8, Forming-8, Flow-8, Sheets-8, Fluids-8, Liquids-8, Stability-8, Turbulence-9, Aerodynamics-9, Dynamics-9, Mechanics (Not Personnel)-9, Physics-9, Holes-9, Coalescence-9, Nozzles-10, Sprayers-10.

Abstract: The three types of instability manifested in flat liquid sheets produced by fan spray nozzles, and their effects on drop size are discussed. Recent analyses of aerodynamic instability are reviewed, experimentally tested, and extended to provide a drop size expression. The conditions for, and the properties of, instability as a result of turbulence are described.

The aerodynamically stable sheet in subatmospheric surroundings disintegrates by the coalescence of perforations. The growth rate, the causation, and the effect of these perforations on drop size are considered. Drop size results at subatmospheric ambient conditions show the dependence of drop size on mode of disintegration.

Reference: Fraser, R. P., Paul Eisenklam, Norman Dombrowski, and David Hasson, **A.I.Ch.E. Journal, 8,** No. 5, p. 672 (November, 1962).

Key Words: Thermodynamics-8, Equilibrium-8, Phase Equilibrium-8, Vapor-Liquid Equilibrium-8, Phases-8, Multicomponent-8, Properties (Characteristics)-9, Physical Properties-9, Activities-9, Solubility Parameters-9, Solutions (Mixtures)-9, Mixtures-9, Azeotropes-9, Binary-, Propane-1, Propylene-1, Propadiene-1, Propyne-1, Hydrocarbons-1, Temperature-6, Composition-6, Relative Volatility-7, Equilibrium Stills-10.

Abstract: A study has been made of composition effects on vapor-liquid equilibria for the system propane-propylene-propadiene-propyne at 50°, 90°, and 140°F. A six-stage equilibrium unit was used in order to minimize the effects of analytical uncertainties. Two or three solutes were studied simultaneously, each at 1 to 3% concentration. The overall composition was varied from nearly pure propane to nearly pure propadiene. Activity coefficients were correlated with the Hildebrand-Scatchard relationships derived for regular solutions. The solubility parameters for the three components other than propane were determined from experimental relative volatility data.

Reference: Hill, Arthur B., Robert H. McCormick, Paul Barton, and M. R. Fenske, **A.I.Ch.E. Journal, 8,** No. 5, p. 681 (November, 1962).

Key Words: A. Absorption-8, Chlorine-1, Water-5, Laminar Jet-10, pH-6, Rate-7, Hydrolysis-9, Penetration Theory-9. B. Kinetics-8, Chlorine-1, Water-1, Hydroxyl Ions-1, Absorption-10, pH-6, Rate-7, Collision-, Transition State-.

Abstract: Chlorine gas is absorbed into aqueous media in a wide range of pH by the jet method. The data is analyzed in accordance with the penetration theory. On this basis the hydrolysis of chlorine is examined in detail and interpreted through the collision and transition state theories.

Reference: Spalding, Charles W., **A.I.Ch.E. Journal, 8,** No. 5, p. 685 (November, 1962).

(Continued on page 716)

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INFORMATION RETRIEVAL

Key Words: Convection-8, Free-, Heat Transfer-8, Transfer-8, Heat-9, Momentum-9, Profiles-9, Temperature-9, Velocity-9, Rates-9, Stability-9, Instability-9, Computers-10, IBM-704-10, Numerical Integration-10, Finite Difference-10, Plates-10, Cylinders-10.

Abstract: Free convection from a vertical isothermal plate is computed by finite difference methods. The transient and steady state velocity field, temperature field, and local heat transfer coefficient are obtained by solving the partial differential equations describing the conservation of mass, momentum, and energy on an IBM-704 computer. The method of solution and problems of instability are described in a previous paper. The results agree with previous solutions for the steady state and short times. The computations are limited to $N_{Pr} = 0.733$.

Natural convection in a fluid in the region inside an infinitely long, horizontal cylinder is computed by finite difference methods. The vertical halves of the cylinder are maintained at different uniform temperatures. The steady state results are for N_{Gr} from 4.5×10^4 to 1.0×10^7 and for N_{Pr} of 0.70 and 10.

Reference: Hellums, J. D., and S. W. Churchill, **A.I.Ch.E. Journal, 8,** No. 5, p. 690 (November, 1962).

these integrals is cumbersome computationally, but machine solution offers little difficulty. This paper is devoted primarily to presentation of some applications of this result.

The velocity with which the particles leave the reactor as a function of particle radius $v(r)$ is the principal problem encountered in the use of this result. It is evident that there is a particle size related to the stokes radius at which the particles will move with the gases. The exact designation of this critical particle size must be left for later work. Furthermore a fraction of particles which are larger than this critical radius will leave by mechanical entrainment. One possible representation of this relationship is

$v(r) = U;\ o \leq r < r_c$

designated as region I (5)

$v(r) = U \dfrac{r_c}{r};\ r_c \leq r < \infty$

designated as region II

Using this velocity-radius relationship one may show that Equation (2) may be written as
in region I

$$\gamma(r) = [K_2\ e^{-K_1 r_c}\ e^{K_1 r}\ r_o^{K_1 r_c}] \int_r^\infty \alpha(\omega)\ \omega^{-K_1 r_c}\ d\omega \qquad (6a)$$

in region II

$$\gamma(r) = K_2 r^{K_1 r_c} \int_r^\infty \alpha(\omega)\omega^{-K_1 r_c}\ d\omega \qquad (6b)$$

Other velocity-particle radius relationships may be used. In particular the effect of size on the rate of mechanical entrainment (the action in region II) needs further elucidation. Lewis and co-workers (2) note only that the entrainment rate "usually decreases with increasing particle size and particle density, although at high gas velocity these trends are reversed." Since the results of this paper are clearly designed for particulate fluidized beds, and since in any operation it is desired to provide a sufficient freeboard to minimize mechanical entrainment, this is not thought to be a limitation on the value of the proposed results. It may be desirable to provide a multiplying constant in the relationship proposed for region II, but this will have no effect on the result.

It is now necessary to have an expression for the feed particle size distribution or to have actual experimental data with which to evaluate the integrals in Equations (6*a*) and (6*b*). If the feed particle size distribution is not badly skewed, the following expression will cover a wide variety of problems, namely

$$\alpha(r) = \frac{0.594}{\zeta_s r_o^4} \left(\frac{r}{r_o}\right)^3 e^{-1.5 \left(\frac{r}{r_o}\right)^2} \qquad (7)$$

wherein r_o represents the particle size at which $\dfrac{d\alpha(r)}{dr} = 0.$

For convenience now let

$$K_2 = \frac{M_F}{V \overline{R}(r)} = \text{constant}$$

$$K_1 = \frac{A_e U}{V \overline{R}(r)} = \text{constant}$$

$$K = K_1 r_c \quad \therefore \quad V \overline{R}(r) = \frac{A_e U r_o}{K}$$

$$X = \frac{r}{r_o}$$

and

$$\Gamma' = \left(\frac{\zeta_s r_o^3 r_c A_e U}{0.594\ M_F}\right) \gamma(r)$$

where Equations (6*a*) and (6*b*) become in region I

$$\Gamma' = K e^{K(4.2X - 2.435)} \int_X^\infty X^{8-K} e^{-1.5 X^2} dX \qquad (8a)$$

in region II

$$\Gamma' = KX^K \int_X^\infty X^{3-K} e^{-1.5X^2} dX \qquad (8b)$$

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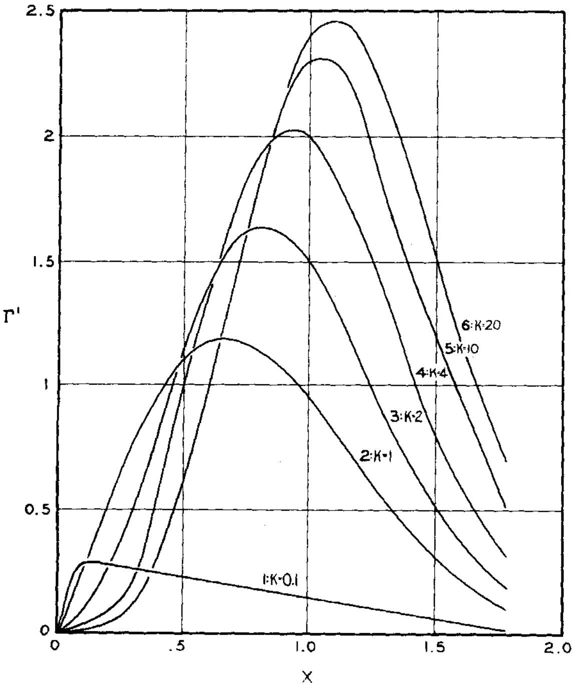

Fig. 2. Steady state bed particle size distributions computed from Equations (8*a*) and (8 *b*).

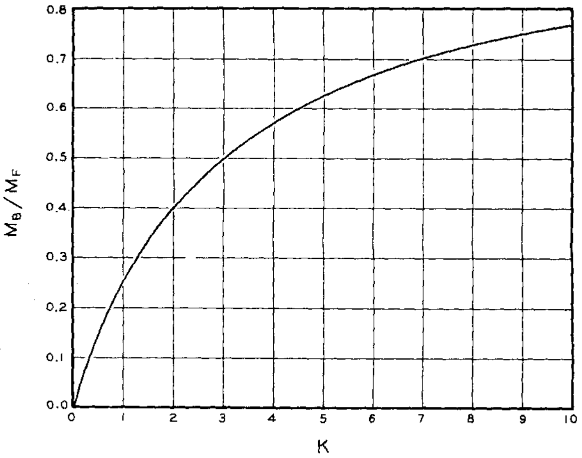

Fig. 3. The ratio of blow over to feed rate as a function of K.

(Continued from page 716)

where Equation (8*a*) is used for $X \leqq 0.238$ and Equation (8*b*) is used for $X \geqq 0.238$.

This result, Equations (8*a*) and (8*b*), was used to prepare Figure 2 by machine computation of the integrals. On this figure the steady state bed particle size distribution function is shown with parameters of *K*. The effect of various operating conditions on bed particle size is available through the functional relationships defining *K* and Γ'. Thus a known bed particle size distribution obtained by sampling from an operating bed may be used to obtain information on the chemical or physical rates which are operable, or, if these are known, then the steady state bed particle size distribution may be predicted.

For example consider the data of McKewan (5) for the reduction of iron ore. The following values are taken or computed from the data presented in this reference:

$$\zeta_s = 5.0 \text{ g./cc.}$$

$$\overline{R}(r)\,[1 \text{ atm., } 500\,°\text{C.}] =$$

$$0.233 \text{ mm./sec.}$$

For a fluidized bed reduction process the following would represent one reasonable set of operating conditions:

U = 1 ft./sec.
r_c = 0.05 mm.
r_o = 0.21 mm. [n 65 mesh]
V = 1 cu. ft.
A_e = 1 sq. ft.
M_F = 120 g./sec.

From these values one may show that

$$K = 0.21$$

$$\Gamma' = 0.80\gamma(r)$$

Thus if a fluidized bed operating under these conditions were fed continuously with a feed of particle size distribution approximating that given by Equation (7), the steady state particle size distribution within the bed would be predicted to be similar to that shown as curve number 1 on Figure 2 with

$$\gamma(r) = 1.25\Gamma'(r)$$

THE BLOW OVER RATE AND THE PARTICLE SIZE DISTRIBUTION IN THE BLOW OVER

Since the steady state particle size distribution in the bed is known, one may write

$$-\gamma(r)\frac{dm}{dt} = -\,4\pi r^2\,\zeta_s\,\gamma(r)V\,\overline{dr/dt} =$$

volume average rate of decrease of mass due to reaction of particles in the size range *r*, *r* + *dr*

Integration over all particle radii yields, at steady state, the difference between the rate of solids feed to the reactor and the blow over rate; thus

$$M_F - M_B =$$

$$\int_0^\infty 4\pi r^2\,\zeta_s\,\gamma(r)V\,\overline{R(r)}\;dr \qquad (9)$$

This result may be simplified, with the definitions in this paper used, to yield

$$\frac{M_B}{M_F} = 1 - \frac{7.46}{K}\int_0^\infty X^2\,\Gamma(x)\;dx \qquad (10)$$

The blow over rate is thus available, again as a function of operating conditions implicit in *K* and Γ'.

Using this result together with the result from equating the number of particles in size range *r*, *r* + *dr* leaving the reactor in time $\delta(t)$ to the particle size at the exit multiplied by the exit area, velocity, and time interval, namely

$$M_B\beta(r)\;dr\;\delta(t) =$$

$$\gamma(r)\;dr\,A_e\,v(r)\;\delta(t) \qquad (11)$$

one may calculate the particle size distribution in the blow over to be

$$\beta = \frac{\gamma(r)A_e v(r)}{M_B}$$

Figure 3 is a plot of M_B/M_F vs. *K* obtained by machine evaluation of Equation (10) from Equations (8*a*) and (8*b*).

SUMMARY AND CONCLUSIONS

In this paper a black box model for blow over rates from a fluidized bed chemical reactor is presented. The model results in a partial differential equation which may be used to evaluate the steady state bed particle size distribution if the inputs and operating conditions are known. Alternately, since the reaction rate appears as one of the parameters in the analysis, measurements of the steady state bed particle size distribution may be used to calculate the reaction rate.

Of probably greater interest is the fact that the blow over rate may be computed by the analysis presented in this paper. Experimentally values of the ratio of blow over to feed rate are easily obtained from an existing plant or pilot plant unit. Such data together with Figure 3 may be used to obtain values of the dimensionless group *K*. This group, used first in Equations (8*a*) and (8*b*), may be used to obtain values for the reaction rate directly. The value of K may then be used to define the steady state bed particle size distribution. This technique has been applied with some success.

ACKNOWLEDGMENT

This problem was first suggested to the author by personnel of the American Cyanamid Company. For the suggestions and interest of these persons, particularly for the help of Dr. Stanley Katz of that company, the author is very grateful. The author is also grateful to the Computation Center of The Johns Hopkins University, most particularly to Miss Patricia Powers, for their assistance.

NOTATION

A = bed cross-sectional area
A_e = cross-sectional area of exit
K_1, K_2 = constants defined in Equation (2)
K = constant = K, r_c
L = bed height
M_F = solids feed rate to bed
M_B = blow over rate
r = particle radius (subscript zero denotes value at maximum in particle size distribution of feed)
r_c = reference or critical particle radius just carried by velocity
$\overline{R}$ = surface reaction rate, that is rate of change of radius with time. The overbar denotes a volume average of this quantity
t = time
$v(r)$ = velocity with which particles of size *r* leave the bed
U = velocity of gas through bed exit port
V = bed volume
X = dimensionless radius, r/r_o
Z = axial distance through bed

Greek Letters

$\alpha(r)$ = particle size distribution function for feed
$\alpha(r)dr$ = number or mass of particles in the feed having feed in the range *r*, *r* + *dr* per unit mass of feed
$\beta(r)$ = particle size distribution function for blow over
$\beta(r)dr$ = number or mass of particles in the blow over having size in the range *r*, *r* + *dr* per unit mass of blow over
$\gamma(r)$ = particle size distribution function in bed, the steady state value
$\gamma(r)dr\,AdZ$ = number or mass of particles of size *r*, *r* + *dr* (in a reactor volume element *AdZ*) per unit volume of the reactor
δ = increment
ζ_s = solids density
Γ' = modified particle size distribution
ω = variable of integration
ζ = variable of integration

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